

A Model for Torsional Analysis of Lattice Towers under the action of Torque

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ABSTRACT

Most towers or masts in practice are used as a more economic option for solid poles as greater heights are required in providing services such as supporting transmission equipments for radio stations, internet providers, wind turbines, telecommunication transceiver equipment and high-voltage electrical transmission. In this paper the torsional analysis of self-supporting lattice towers will be considered even though the methodology can be applied similarly in the analysis of masts. The failure or collapse of self-supporting lattice towers can be very devastating and result in service loss or down time as well as loss of valuable lives and properties which could most often happen without notice and in a flash. Proper analyses of such structures are therefore a requirement not an option.

In view of the rigorous and cumbersome calculations involved in the analysis of self-supporting lattice towers used in telecommunications and power distribution, it is necessary to propose a simplified model for analysis of self-supporting lattice towers that can give acceptable results when compared to methods currently in use. The major forces requiring analysis in any such serviceable structure to continue maintaining its state of static and dynamic equilibrium are Normal Forces (Tension or Compression), Shear Forces, Flexure, Moment and Torque. This model in a previous research had been used to determine the Critical Buckling load and Natural frequency of vibration of a self-supporting lattice tower and we hope to apply it in determining the angle in torsion due to torque.

This paper as earlier discussed proposes a model for the determination of the angle of twist for self-supporting lattice towers subjected to torsion as a result of torque developed due to the action of wind loads or other external forces which could be determined with the expression $\sum_{i=1}^N \frac{P_i \bar{P}_i}{EA_i}$ though more cumbersome. The proposed model idealizes the self-supporting lattice tower as an equivalent non-prismatic solid beam-column structure restrained at its base being of the same height and similar shape as the self-supporting lattice tower whose cross-sectional dimensions are likely to differ. The expressions $\tau_{max} = \frac{2\sqrt{2}TLB_0}{3(b-B_0)(b^2-B_0^2)}$ and $\theta = \frac{\sqrt{2}\tau L}{Gb}$ are proposed by the use of the beam-column model for the computation of the angle of twist for self-supporting lattice towers under the action of torque. The equivalent beam-column structure will have dimensions of b at its free end and B_0 at its restrained base.

The model for this research was developed as a result of a proposed theory which states that “Any lattice Tower under the action of forces can be analysed using an equivalent solid beam-column having the same height, shape and value of deflection or angle of torsion along its length as the lattice tower though having different cross sectional dimensions as long as the same value of forces are applied at the same point and in the same direction on the beam-column model”.

Keywords: Beam-column, Tower Model, Torsion, Torsional Angle, Torque, Truss.

1.0 INTRODUCTION

Lattice or truss towers are tall steel frame or truss structures made up of smaller member structures used for different purposes such as installation of equipment for telecommunication, radio transmission, satellite reception, air traffic control, television transmission, power transmission, wind turbines, flood lights, meteorological measurements, etc. The analyses of such structures are very necessary since the outcome is important to safe guard lives, properties and investments. The major forces acting on lattice towers are wind loads, equipment and self weight. The self weight normally results in vertical loading on the entire structure which in turn develops tensile and compressive forces in the members. The extent of vibration or torsion developed in the structure depends on the type of equipment places on it, the expected use and wind speed and direction.

Analyzing such structures requires a lot of rigorous and cumbersome calculations which could result to time loss

and discourage analysis of such natures. In attempting to offer a solution to such situation, N. N. Osadebe, M. E. Onyia and M. C. Nwosu in their research successfully developed a representative model with an expression which makes it possible to **determine the natural vibration frequencies of self-supporting lattice towers** using the developed model and expression.

M. E. Onyia and M. C. Nwosu in a further research successfully developed an expression using the exact same model used in the previous research to successfully determine the critical buckling load of self-supporting lattice towers. The research of such nature conducted with the beam-column model so far can only for now **determine the natural vibration frequencies and critical buckling load** of self supporting lattice towers. The use of the model for torsional analysis was not yet tested.

In attempting to provide a less cumbersome yet accurate method of determining the angle of twist for a lattice tower

under the action of torque, J. J. Cao, A. J. Bell and K. J. Xu developed an expression $\theta = \sum_{i=1}^N \frac{P_i \bar{P}_i}{EA_i} l_i$ to successfully carry out a direct torsional analysis of lattice towers. The research by L. L. Cao et al. though impressive is still cumbersome in nature having different expressions for different shapes and member arrangements of the same lattice tower. The equivalent beam-column method does not consider the member arrangements but rather the internal forces in the lattice members. This prompted the desire for a further research into the use of the equivalent beam-column method in torsional analysis of lattice tower. In the paper we will focus more on a 4-legged self-supporting lattice tower.

In the torsion analysis of a self-supporting lattice tower, we are more interested in the extent of twist the structure undergoes under the action of the applied torque which can be determined by using the value of internal forces developed in the truss members since the failure of any or all of the truss members that make up a lattice tower either in tension, compression or bending could lead to the gradual or immediate failure or collapse of the entire structure. If the lattice tower is twisted to a point where the forces acting on the structure is no longer in equilibrium, then there is a likelihood of failure or collapse.

The ideology deployed in determining the angle of twist of lattice tower in this paper is based on the fact that if the same value of torque expected to act on a self-supporting tower is applied at the same point on the equivalent beam-column structure (model), then the value for angle of twist obtained from the equivalent beam-column structure (model) should be equal to that expected for the self-supporting lattice tower.

In determining the angle of twist for a solid beam-column structure, the expression for that is readily available is that for a cylindrical structure, but in this paper we are more interested in the analysis of a non-prismatic solid beam-column with square cross section having a linearly tapering width dimension along its length as shown in figure 2.

Hasseb Ahmed Khan in a Master's research at University of Karachi in 1966 developed expressions for the torsional analysis of a prismatic structure with square cross section having width dimension $2a$.

$$\tau_{max} = 1.351Ga\theta$$

$$T = 0.1406G\theta(2a)^4$$

Where,

T is Torque applied on the structure.

τ_{max} is the maximum value for shear stress.

G is the shear modulus of the material (in this case steel).

$2a$ is the dimension of the width.

θ is the angle of twist.

The expression above can be re-written thus;

$$\tau_{max} = \frac{T}{1.6651a^3} \quad (1)$$

$$\theta = \frac{\tau}{1.351Ga} \quad (2)$$

The expressions were derived with a prismatic beam-column having constant width throughout its length. The expression has to be adapted for a non-prismatic beam-column with variable width when $2a$ will be replaced with the term B_x .

Jan Francü et al. in their research titled "Torsion of a Non-Circular Bar" also derived an expression for determining the value of angle of twist for a rectangular cross section structure having the expression for polar moment of inertia, J ;

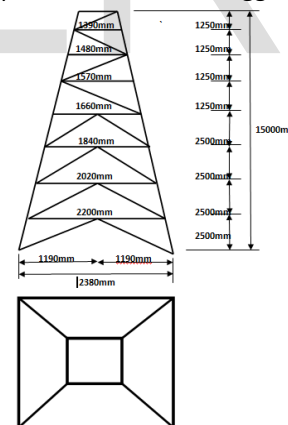
$$J = k_1 \left(\frac{a}{b}\right) ab^3$$

Where, constant $k_1 = 0.141$.

a and b are the dimensions of rectangular the cross section.

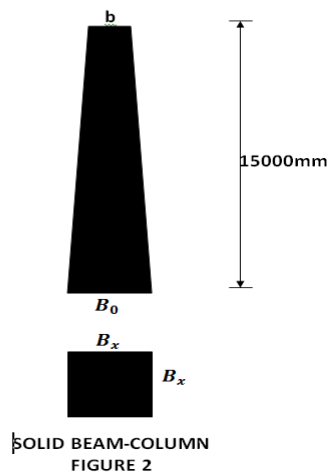
2.0 METHODOLOGY

In view of the discussions so far, it is expedient to explain the method employed in this research. The exact same tower analysed in the research by N. N. Osadebe et al. as shown in figure 1 having top and base widths of 1.30 meters and 2.38 meters respectively and tower height of 15 meters with an external of 1000KN horizontally applied at the top of the lattice Four-legged tower.



4-legged self-support lattice tower
Figure 1

When the same value of force was applied on the equivalent beam-column with variable width B_x as shown in figure 2 and the horizontal deflections of both the lattice tower and the beam-column taken to be exactly the same, the top and base width dimensions of the beam-column was found to be $b = 0.2569$ meters and $B_0 = 0.750852$ meters respectively as referenced from the research of Osadebe N. N. et al.



Having determined the values of the top and base width for the equivalent beam-column, a torque of value 650KNm was applied at the top of the four-legged lattice tower and similarly the same value of torque was applied at the top of the equivalent beam-column and the values obtained for the angle of twist for both the four-legged lattice tower and the equivalent beam-column were calculated, tabulated and compared.

2.1 MATHEMATICAL MODELLING

In analyzing the effect of the applied torque on the lattice tower and beam-column there was the need to derive the expressions for determination of the angle of twist for the lattice tower and beam-column. It is assumed that the torque acts in the horizontal direction with respect to the structures and produces a purely horizontal twist. The lattice tower is not expected to have vertical reactions due to the action of Torque.

2.1.1 Expression of Angle of Twist for 4-legged self-supporting lattice tower

We will recall that the expression for the determination of deflection or horizontal displacement for a self-supporting lattice tower when a horizontal force is applied at the top is;

$$\Delta = \sum_{i=1}^N \frac{F_i \bar{F}_i}{EA_i} l_i.$$

Where,

F is the value of the internal force in the member i under the applied force.

\bar{F} is the value of the internal force in the member i under the action of unit force ($F = 1$).

A_i and l_i are the cross sectional area and length of the member i .

N is the number of members in the lattice tower.

E is the young's modulus of elasticity.

In the same way, the expression for determination of angle of twist of lattice tower is;

$$\theta = \sum_{i=1}^N \frac{P_i \bar{P}_i}{EA_i} l_i \quad (3)$$

Where,

P is the value of the internal force in the member i under the action of torque T .

\bar{P} is the value of the internal force in the member i under the action of unit torque ($T = 1$).

A_i and l_i are the cross sectional area and length of the member i .

N is the number of members in the lattice tower.

E is the young's modulus of elasticity.

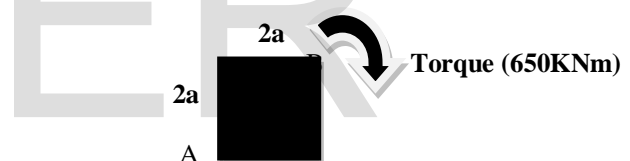
The above equation was referenced from that derived by J. J. Cao et al. on page 336 of their publication titled "Torsional Analysis of Lattice Towers", for the determination of angle of twist for lattice towers. Note that the expressions for deflection and angle of twist for the lattice tower are the same except for the fact that the values of the internal member forces differs and depends on the type of force applied. The expression was used in the determination of angle of twist for the self-supporting tower.

2.1.2 Expression of Angle of Twist for column-beam

In deriving an expression for determination of angle of twist for a beam-column structure, we applied the Saint Venant hypothesis. The expressions were of two types, that which was adapted from an expression for prismatic beam-columns in equations 1 & 2 and that derived from first principle, taking the shape of the beam-column into consideration.

Method 1:

From equations 1 & 2,



Beam-Column Cross Section

Figure 3a

The expressions from equations (1) and (2);

$$\tau = \frac{T}{1.6651a^3}$$

$$\theta = \frac{\tau}{1.351Ga}$$

Which were derived for prismatic beam-column was adapted for use in the determination of the value of angle of twist for non-prismatic solid beam-column structure as shown in figure 3a.

In adapting the expression;

B_x was taken as the width dimension which changes linearly with the value of x which varies from 0 to L .

$$B_x = B_0 + (b - B_0) x/L$$

In this case $B_x = 2a$

Therefore, $a = B_x/2$.

The maximum shear τ_{max} along the length of the beam-column is expected to occur at the restricted base of the beam-column structure $x = 0$, while the maximum value for the angle of twist θ is expected to occur at the top of the Non-prismatic beam-column structure $x = L$.

At

$$x = 0, \quad B_x = B_0, \quad \text{Shear Stress } \tau \text{ is maximum.}$$

$$\text{At } x = L, \quad B_x = b, \quad \text{Angle of Twist } \theta \text{ is maximum.}$$

The expressions above were then written as;

$$\text{Shear Stress, } \tau = \int_0^L \frac{4.80452 T}{B_0^3}$$

Integrating with respect to x ,

$$\tau = \frac{2.40226 TL}{(b-B_0)(B_0^2-b^2)} \quad (4)$$

Also,

$$\text{Angle of Twist, } \theta = \int_0^L \frac{1.4804 \tau A_s}{Gb}$$

$$A_s = \frac{(b+B_0)}{2}$$

Integrating with respect to x ,

$$\theta = \frac{\tau L(b+B_0)(\ln b)}{1.351(b-B_0)G} \quad (5)$$

Where, B_0 is the width at the base where shear stress is maximum.

b is the width at the top of the beam-column where twist angle is maximum.

A_s is the surface area of the tapered structure by which the expression is multiplied to take care of the shape factor that was not considered in the derivation of the expression.

Method 2:

The second expression for the angle of twist of a solid beam-column structure with linearly varying width along its length was derived considering the linearly tapering shape of the width of the structure.

The expression for the angle of twist for a solid cylinder under the action of torque is

Where,

$$\tau = \frac{Tr}{J} \quad (6)$$

$$\theta = \frac{TL}{GJ} = \frac{\tau L}{Gr} \quad (7)$$

Recall that the structure under study being a four-legged lattice tower should be have a representative model in a solid beam-column with a square cross section which

cannot be represented by a cylinder with a circular cross section.

In calculating the angle of twist for the beam-column structure, we had to consider the properties and shape of the equivalent beam-column structure. Figure 3b shows the view of the beam-column structure when looked at from the top.



Beam-Column Cross Section
Figure 3b

$$\text{Polar moment of Inertia, } J_x = \iint r_x^2 dA_x \quad (8)$$

$$\text{Torque Distance of action, } r_x = \sqrt{(B_{rx}/2)^2 + (B_{rx}/2)^2}$$

$$r_x = B_{rx}/\sqrt{2} \quad (9)$$

$$\text{At the bottom of the beam-column, } r_0 = B_0/\sqrt{2}$$

$$\text{At the top of the beam-column, } r_L = b/\sqrt{2}$$

$$\text{Beam - Column Width, } B_x = [B_0 + (b - B_0)x/L]$$

$$\text{Beam - Column Area, } A_x = B_x^2$$

$$dA_x = 2B_x dB_x$$

$$J_x = \int B_x^3 dB_x$$

Integrating with respect to B_x ;

$$J_x = B_x^4/4 \quad (10)$$

From equation 3;

$$\tau_x = \int_0^L \frac{Tr_x dx}{J_x} = \int_0^L \frac{4TB_{rx} dx}{\sqrt{2}B_x^4}$$

Integrating with respect to x ;

$$\tau_x = \frac{2\sqrt{2}TLB_{rx}}{3(b-B_0)B_x^3} + C$$

Considering that the boundaries $x = L$ and $x = 0$;

$$\tau = \frac{2\sqrt{2}TLB_{rx}}{3(b-B_0)(b^3-B_0^3)} \quad (11)$$

From equation 2;

$$\theta_x = \int_0^L \frac{TL}{GJ_x} dx$$

$$T = \frac{3\tau(b - B_0)(b^3 - B_0^3)}{2\sqrt{2}LB_{rx}}$$

$$\theta_x = \int_0^L \frac{TL}{GJ_x} dx$$

Substituting the expression for T , J_x and integrating with respect to x ;

$$\theta_x = \frac{\sqrt{2}\tau L}{GB_{rx}} + C$$

$$\theta_x = \frac{12(b - B_0)(b^3 - B_0^3)\tau L}{6\sqrt{2}(b - B_0)(b^3 - B_0^3)GB_{rx}}$$

Considering that the boundaries $x = L$ and $x = 0$;

The required angle of twist should occur at the top of the beam-column;

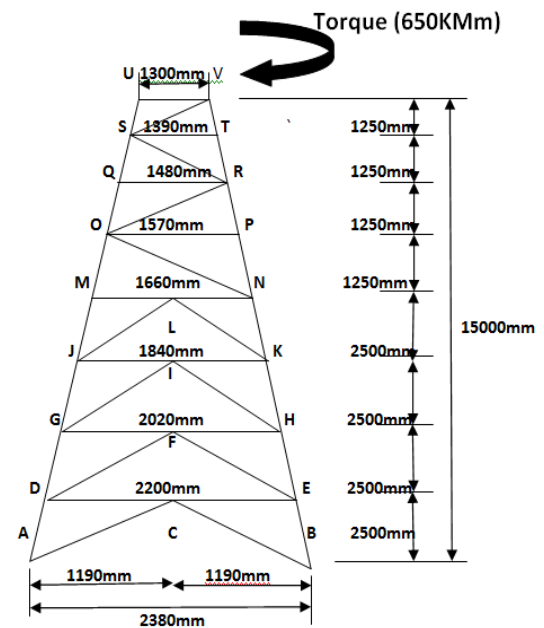
$$\theta = \frac{\sqrt{2}\tau L}{GB_{rx}} = \frac{\sqrt{2}\tau L}{Gb} \quad (12)$$

3.0 PRESENTATION OF RESULTS

In order to compare the values for the angle of twist obtained for the self-supporting lattice tower and the equivalent beam-column as shown in figures 3 and 4 respectively, the two structures were subjected to a torque of 650KNm torque in the same direction at the free ends of the individual structures.

The self-supporting lattice tower has the following properties;

- (i) A 4-legged self-supporting lattice tower.
- (ii) Truss members with total volume, $V_T = 0.9684\text{m}^3$.
- (iii) Density = 7850kg/m
- (iv) Young's Modulus, $E = 210 \times 10^6 \text{ KN/m}^2$.
- (i) Dimension at the free end (top), $b_T = 1.30\text{m}$.
- (v) Dimension at the base, $B_T = 2.38\text{m}$.
- (vi) Height of Lattice Tower, $L_T = 15\text{m}$.
- (vii) The Lattice Tower varies linearly along its height.



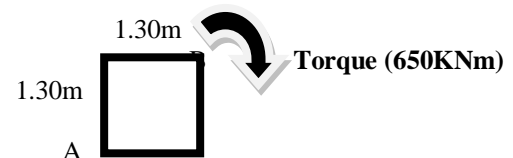
4-LEGGED SELF-SUPPORT LATTICE TOWER
FIGURE 4

The equivalent beam-column has the following properties;

- (ii) Solid homogenous structure with square cross section.
- (iii) Density = 7850kg/m.
- (iv) Young's Modulus, $E = 210 \times 10^6 \text{ KN/m}^2$.
- (v) Dimension at the free end (top), $b = 0.2569 \text{ m}$.
- (vi) Dimension at the base, $B_0 = 0.2569 \text{ m}$.
- (viii) Height of the beam-column = 15.0m
- (vii) The beam-column varies linearly along its height.

3.1. Calculation for Angle of Twist of Lattice Tower.

In calculating the angle of twist for the self-supporting lattice tower, we needed to consider the properties and shape of the 4-legged lattice tower structure. Figure 5 shows the shape of the lattice tower when looked at from the top.



Lattice Tower Cross Section
Figure 5

The Torque acts at the mid-point between points A and B of the lattice tower section and the torque will normally be resisted horizontally at the fixed base of the tower and note that since the torque acts horizontally, the internal force of the lattice tower legs or vertical members should be equal to zero.

Similar to the determination of expression in equation 9;

$$\text{The distance of action of torque, } r = \sqrt{0.65^2 + 0.65^2} = 0.919238816 \text{ m.}$$

The value of force being resisted by the each leg of the 4-legged self-supporting lattice tower,

$$F = 650 / (4 \times 0.919238816) = 176.78 \text{ KN.}$$

Table 1 shows the result of two the dimensional analysis for one face of the 4-legged self-support lattice tower which shows the internal forces within the individual tower members when a torque of 650KNm was applied and also the value of member internal forces when a unit torque of 1KNm was applied at the same point on the lattice tower.

Table 2 shows the value for the angle of twist for each member and also the summation of the angle of twists determined in analyzing the lattice tower. From the table 2, Recall that from equation 3, $\theta = \sum_{i=1}^N \frac{P_i \bar{P}_i}{A_i E}$
Angle of twist per side, $\theta_{s1} = 0.008957575$

Considering the four sides of the 4-legged lattice tower, the total angle of twist which occurs at the top of the lattice tower,

$$\theta = 4 \times 0.008957575$$

$$\theta = 0.0358303.$$

TABLE 1: LATTICE TOWER TORQUE MEMBER FORCES.

S/N	MEMBER	TOWER HEIGHT AT JOINT (m)	MEMBER INTERNAL FORCE (KN)	MEMBER INTERNAL FORCE FOR UNIT TORQUE (KN)	MEMBER ANGLE DEGREES	MEMBER ANGLE RADIANS	MEMBER LENGTH (m)	SECTION WIDTH (B) (m)	DISTANCE OF TORQUE ACTION (Y) (m)
1	UV	15	176.7766953	0.271964147	0	0	1.3	1.3	0.919238816
2	US		0	0	267.94	4.676446	1.251	0	0
3	VS		-241.7136814	-0.371867202	223	3.892093	1.836	1.3	0.919238816
4	VT		0	0	272.06	4.748354	1.251	0	0
5	ST	13.75	165.3307222	0.254354957	0	0	1.39	1.39	0.982878426
6	SR		219.0628759	0.337019809	319	5.567813	1.903	1.39	0.982878426
7	TR		0	0	272.06	4.748354	1.251	0	0
8	SQ		0	0	267.94	4.676446	1.251	0	0
9	QR	12.5	155.2768269	0.238887426	0	0	1.48	1.48	1.046518036
10	QO		0	0	267.94	4.676446	1.251	0	0
11	RO		-212.3160715	-0.32664011	223	3.892093	1.972	1.48	1.046518036
12	RP		0	0	272.06	4.748354	1.251	0	0
13	OP	11.25	146.3756076	0.225193242	0	0	1.57	1.57	1.110157646
14	ON		185.0970932	0.284764759	322.26	5.624511	2.042	1.57	1.110157646
15	PN		0	0	272.06	4.748354	1.251	0	0
16	OM		0	0	267.94	4.676446	1.251	0	0
17	ML	10	138.4395807	0.21298397	0	0	0.83	1.66	1.173797257
18	NL	10	138.4395807	0.21298397	180	3.1416	0.83	1.66	1.173797257
19	MJ		0	0	267.94	4.676446	2.502	0	0
20	NK		0	0	272.06	4.748354	2.502	0	0
21	LI		-200.4693585	-0.308414398	249.8	4.359843	2.664	1.66	1.173797257
22	LK		200.4573506	0.308395924	290.2	5.064957	2.664	1.66	1.173797257
23	JI	7.5	124.8965782	0.192148582	0	0	0.92	1.84	1.301076477
24	KI	7.5	124.8965782	0.192148582	180	3.1416	0.92	1.84	1.301076477
25	JG		0	0	267.94	4.676446	2.502	0	0
26	KH		0	0	272.06	4.748354	2.502	0	0
27	IG		-166.7078335	-0.25647359	248	4.328427	2.696	1.84	1.301076477
28	IH		166.69874	0.2564596	292	5.096373	2.696	1.84	1.301076477
29	GF	5	113.7671801	0.175026431	0	0	1.01	2.02	1.428355698
30	HF	5	113.7671801	0.175026431	180	3.1416	1.01	2.02	1.428355698
31	GD		0	0	267.94	4.676446	2.502	0	0
32	HE		0	0	272.06	4.748354	2.502	0	0
33	FD		-141.2423511	-0.217295925	246.25	4.297883	2.731	2.02	1.428355698
34	FE		141.2352768	0.217285041	293.75	5.126917	2.731	2.02	1.428355698
35	DC	2.5	104.4589563	0.160706087	0	0	1.1	2.2	1.555634919
36	EC	2.5	104.4589563	0.160706087	180	3.1416	1.1	2.2	1.555634919
37	DA		0	0	267.94	4.676446	2.502	0	0
38	EB		0	0	272.06	4.748354	2.502	0	0
39	CA		-121.5446687	-0.186991798	244.55	4.268213	2.769	2.2	1.555634919
40	CB		121.5390401	0.186983139	295.45	5.156587	2.769	2.2	1.555634919

TABLE 2: LATTICE TOWER ANGLE OF TWIST.

S/N	MEMBER	TOWER HEIGHT AT JOINT (m)	MEMBER INTERNAL FORCE (KN)	MEMBER INTERNAL FORCE FOR UNIT TORQUE (KN)	MEMBER LENGTH (m)	MEMBER AREA (m ²)	YOUNG'S MODULUS (E) (KN/m ²)	MEMBER VOLUME (m ³)	ANGLE OF ROTATION FOR 4 SIDES OF TOWER
1	UV	15	176.7766953	0.271964147	1.3	0.000225	210000000	0.0002925	0.005291005
2	US		0	0	1.251	0.000225	210000000	0.000281475	0
3	VS		-241.7136814	-0.371867202	1.836	0.000225	210000000	0.0004131	0.013970758
4	VT		0	0	1.251	0.000225	210000000	0.000281475	0
5	ST	13.75	165.3307222	0.254354957	1.39	0.000225	210000000	0.00031275	0.004948422
6	SR		219.0628759	0.337019809	1.903	0.000744	210000000	0.001415832	0.00359692
7	TR		0	0	1.251	0.000744	210000000	0.000930744	0
8	SQ		0	0	1.251	0.000744	210000000	0.000930744	0
9	QR	12.5	155.2768269	0.238887426	1.48	0.000744	210000000	0.00110112	0.001405495
10	QO		0	0	1.251	0.000744	210000000	0.000930744	0
11	RO		-212.3160715	-0.32664011	1.972	0.00151	210000000	0.00297772	0.001725135
12	RP		0	0	1.251	0.0028	210000000	0.0035028	0
13	OP	11.25	146.3756076	0.225193242	1.57	0.0028	210000000	0.004396	0.000352052
14	ON		185.0970932	0.284764759	2.042	0.0028	210000000	0.0057176	0.000732191
15	PN		0	0	1.251	0.0028	210000000	0.0035028	0
16	OM		0	0	1.251	0.00512	210000000	0.00640512	0
17	ML	10	138.4395807	0.21298397	0.83	0.00512	210000000	0.0042496	9.1045E-05
18	NL	10	138.4395807	0.21298397	0.83	0.00512	210000000	0.0042496	9.1045E-05
19	MJ		0	0	2.502	0.00512	210000000	0.01281024	0
20	NK		0	0	2.502	0.00512	210000000	0.01281024	0
21	LI		-200.4693585	-0.308414398	2.664	0.00512	210000000	0.01363968	0.000612736
22	LK		200.4573506	0.308395924	2.664	0.00512	210000000	0.01363968	0.000612685
23	JI	7.5	124.8965782	0.192148582	0.92	0.00512	210000000	0.0047104	8.21384E-05
24	KI	7.5	124.8965782	0.192148582	0.92	0.00512	210000000	0.0047104	8.21384E-05
25	JG		0	0	2.502	0.00512	210000000	0.01281024	0
26	KH		0	0	2.502	0.00512	210000000	0.01281024	0
27	IG		-166.7078335	-0.25647359	2.696	0.00512	210000000	0.01380352	0.000428834
28	IH		166.69874	0.2564596	2.696	0.00512	210000000	0.01380352	0.000428787
29	GF	5	113.7671801	0.175026431	1.01	0.00512	210000000	0.0051712	7.48191E-05
30	HF	5	113.7671801	0.175026431	1.01	0.00512	210000000	0.0051712	7.48191E-05
31	GD		0	0	2.502	0.00808	210000000	0.02021616	0
32	HE		0	0	2.502	0.00808	210000000	0.02021616	0
33	FD		-141.2423511	-0.217295925	2.731	0.00512	210000000	0.01398272	0.000311824
34	FE		141.2352768	0.217285041	2.731	0.00512	210000000	0.01398272	0.000311792
35	DC	2.5	104.4589563	0.160706087	1.1	0.00512	210000000	0.005632	6.86976E-05
36	EC	2.5	104.4589563	0.160706087	1.1	0.00512	210000000	0.005632	6.86976E-05
37	DA		0	0	2.502	0.00808	210000000	0.02021616	0
38	EB		0	0	2.502	0.00808	210000000	0.02021616	0
39	CA		-121.5446687	-0.186991798	2.769	0.00512	210000000	0.01417728	0.000234127
40	CB		121.5390401	0.186983139	2.769	0.00512	210000000	0.01417728	0.000234106

3.2. Calculation for Angle of Twist for the Solid Beam-Column Structure.

METHOD 1:

From equations 4 and 5,

$$\tau = \frac{2.40226 TL}{(b-B_0)(B_0^2-b^2)}$$

$$\theta = \frac{\tau L(b+B_0)(\ln b)}{1.351(b-B_0)G}$$

Where, $b = 0.2569m$ and $B_0 = 0.750852m$

$$\tau_{max} = 95285.00032 \text{ KN/m}^2.$$

$$\theta = 0.036587609$$

METHOD 2:

From equations 11 and 12;

$$\tau = \frac{2\sqrt{2} TL B_{rx}}{3(b-B_0)(b^3-B_0^3)}$$

$$\theta = \frac{\sqrt{2}\tau L}{Gb}$$

Where the Poisson ratio, $\mu = 0.31$

Shear modulus, G

= Young's modulus (E)/ $2x(1 + \text{Poisson's Ratio})$

$$G = E/2(1 + \mu).$$

$$G = 80152672 \text{ KNm}^2.$$

This implies that, $\tau = 34386.46 \text{ KNm}^2$

Therefore; $\theta = 0.03542514$

The summary of the results is presented in Table below.

Table 3 – Comparison of Results

Description	Lattice Tower	Beam-Column MTD 1	Beam-Column MTD 2
Top Dimension	1300mm	256.90mm	256.90mm
Base Dimension	2380mm	750.85mm	750.85mm
Torque	650KNm	650KNm	650KNm
Angle of Twist	0.0358303	0.036587609	0.03542514
% Difference	0%	2.11%	1.13%

4.0 CONCLUSIONS

A comparison of the values of the angle of twist for the actual self-supporting lattice tower and the proposed equivalent Beam-column model shows a marginal percentage difference of 1.13% to 2.11%. The proposed model is acceptable since it gives a lower-bound value of the angle of twist, which is a welcome safeguard against

twist. The derived model expressions can also be for less cumbersome analysis modified at ease to analyze lattice towers of different sizes and cross-sectional shapes, such as circular-shaped, triangular-shaped (3-legged) and polygon-shaped towers.

Having previously proven in two earlier researches first by N. N. Osadebe, M. E. Onyia and M. C. Nwosu titled “A Model for the Determination of the Natural Vibration Frequencies of Self-Supporting Lattice Towers” and secondly by M. E. Onyia and M. C. Nwosu titled “A Model for the Determination of the Critical Buckling Load of Self-Supporting Lattice Towers it would be proper to state that **“Any lattice Tower under the action of any kind of forces can be analysed using an equivalent solid beam-column model having the same height, shape and value of deflection or angle of torsion as the lattice tower though having different width dimensions as long as the same value of forces are applied at the same point and in the same direction on the beam-column model”**. The basic short coming of this method of analysis as earlier discussed is in the fact that only limited information is known of the individual lattice members, but further analysis can be done on the individual member elements if required which would not likely be cumbersome.

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